



1. Show that

$$\frac{d}{dx}[\ln(\tanh x)] = 2 \operatorname{cosech} 2x, \quad x > 0. \quad (4)$$

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**Question 1 continued**

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**(Total 4 marks)**

**Q1**

3



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2. Find the values of  $x$  for which

$$8 \cosh x - 4 \sinh x = 13,$$

giving your answers as natural logarithms.

(6)

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3. Show that

$$\int_5^6 \frac{3+x}{\sqrt{x^2-9}} dx = 3 \ln \left( \frac{2+\sqrt{3}}{3} \right) + 3\sqrt{3} - 4.$$

(7)

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**Question 3 continued**

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Lined area for writing answers.

**(Total 7 marks)**

**Q3**

7

**Turn over**











**Question 5 continued**

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Question 5 continued

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(Total 12 marks)

Q5

13

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6.

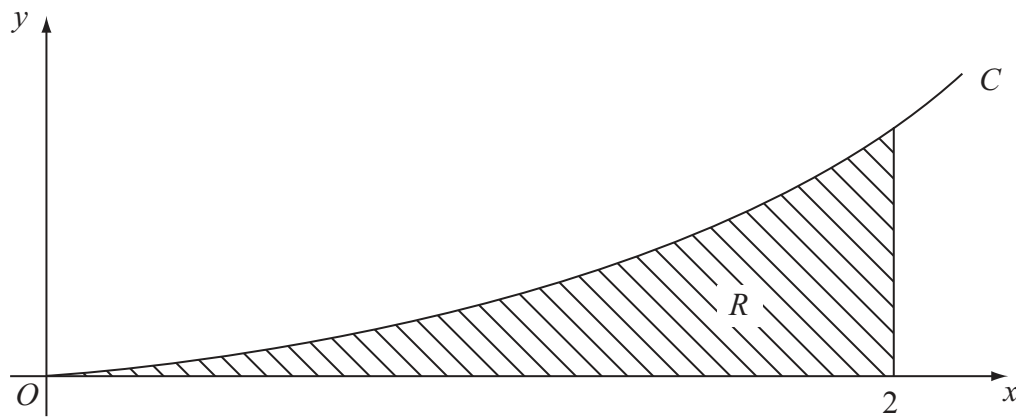


Figure 1

Figure 1 shows the curve  $C$  with equation

$$y = \frac{1}{10} \cosh x \arctan(\sinh x), \quad x \geq 0.$$

The shaded region  $R$  is bounded by  $C$ , the  $x$ -axis and the line  $x = 2$ .

(a) Find  $\int \cosh x \arctan(\sinh x) dx$ . (5)

(b) Hence show that, to 2 significant figures, the area of  $R$  is 0.34 (2)

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**Question 6 continued**

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**(Total 7 marks)**

Q6



7. The hyperbola  $H$  has equation

$$\frac{x^2}{16} - \frac{y^2}{9} = 1.$$

(a) Show that an equation for the normal to  $H$  at a point  $P(4 \sec t, 3 \tan t)$  is

$$4x \sin t + 3y = 25 \tan t. \tag{6}$$

The point  $S$ , which lies on the positive  $x$ -axis, is a focus of  $H$ . Given that  $PS$  is parallel to the  $y$ -axis and that the  $y$ -coordinate of  $P$  is positive,

(b) find the values of the coordinates of  $P$ . (5)

Given that the normal to  $H$  at this point  $P$  intersects the  $x$ -axis at the point  $R$ ,

(c) find the area of triangle  $PRS$ . (3)

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8. The curve  $C$  has parametric equations

$$x = 3(t + \sin t), \quad y = 3(1 - \cos t), \quad 0 \leq t < \pi.$$

(a) Show that  $\frac{dy}{dx} = \tan \frac{t}{2}$ . (3)

The arc length  $s$  of  $C$  is measured from the origin  $O$ .

(b) Show that  $s = 12 \sin \frac{t}{2}$ . (4)

(c) Hence write down the intrinsic equation of  $C$  in the form  $s = f(\psi)$ . (1)

The point  $P$  lies on  $C$  and the arc  $OP$  of  $C$  has length  $L$ . The arc  $OP$  is rotated through  $2\pi$  radians about the  $x$ -axis.

(d) Show that the area of the curved surface generated is given by

$$\frac{\pi L^3}{36}. \quad (7)$$

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